A

Surname _____

Other Names _____

Centre Number _____

Candidate Number _____

Candidate Signature _____

I declare this is my own work.

A-level
FURTHER MATHEMATICS

Paper 2

7367/2

Thursday 4 June 2020 Afternoon

Time allowed: 2 hours

At the top of the page, write your surname and other names, your centre number, your candidate number and add your signature.

[Turn over]

- **You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.**
- **You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)**

INSTRUCTIONS

- **Use black ink or black ball-point pen. Pencil should only be used for drawing.**
- **Answer ALL questions.**
- **You must answer each question in the space provided for that question.**
- **Do NOT write on blank pages.**
- **Show all necessary working; otherwise marks for method may be lost.**
- **Do all rough work in this book. Cross through any work you do not want to be marked.**

INFORMATION

- **The marks for questions are shown in brackets.**
- **The maximum mark for this paper is 100.**

ADVICE

- **Unless stated otherwise, you may quote formulae, without proof, from the booklet.**
- **You do not necessarily need to use all the space provided.**

DO NOT TURN OVER UNTIL TOLD TO DO SO

Answer ALL questions in the spaces provided.

#

1 **Three of the four expressions below are equivalent to each other.**

Which of the four expressions is NOT equivalent to any of the others?

Circle your answer. [1 mark]

$a \times (a + b)$

$(a + b) \times b$

$(a - b) \times b$

$a \times (a - b)$

2 Given that $\arg(a + b\mathrm{i}) = \varphi$, where a and b are positive real numbers and $0 < \varphi < \frac{\pi}{2}$, three of the following four statements are correct.

Which statement is NOT correct?

Tick (✓) ONE box. [1 mark]

☐ $\arg(-a - b\mathrm{i}) = \pi - \varphi$

☐ $\arg(a - b\mathrm{i}) = -\varphi$

☐ $\arg(b + a\mathrm{i}) = \frac{\pi}{2} - \varphi$

☐ $\arg(b - a\mathrm{i}) = \varphi - \frac{\pi}{2}$

3 Find the gradient of the tangent to the curve

$$y = \sin^{-1} x$$

at the point where $x = \frac{1}{5}$

Circle your answer. [1 mark]

$\frac{5\sqrt{6}}{12}$ $\frac{2\sqrt{6}}{5}$ $\frac{4\sqrt{3}}{25}$ $\frac{25}{24}$

4 The matrices A and B are defined as follows:

$$\mathbf{A} = \begin{bmatrix} x+1 & 2 \\ x+2 & -3 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} x-4 & x-2 \\ 0 & -2 \end{bmatrix}$$

Show that there is a value of x for which $\mathbf{AB} = k\mathbf{I}$, where $\mathbf{I}$ is the 2×2 identity matrix and k is an integer to be found. [3 marks]

[Turn over]

5 Solve the inequality

$$\frac{2x + 3}{x - 1} \leq x + 5$$

[5 marks]

[Turn over]

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[Turn over]

6 **Find the sum of all the integers from 1 to 999 inclusive that are not square or cube numbers. [5 marks]**

[Turn over]

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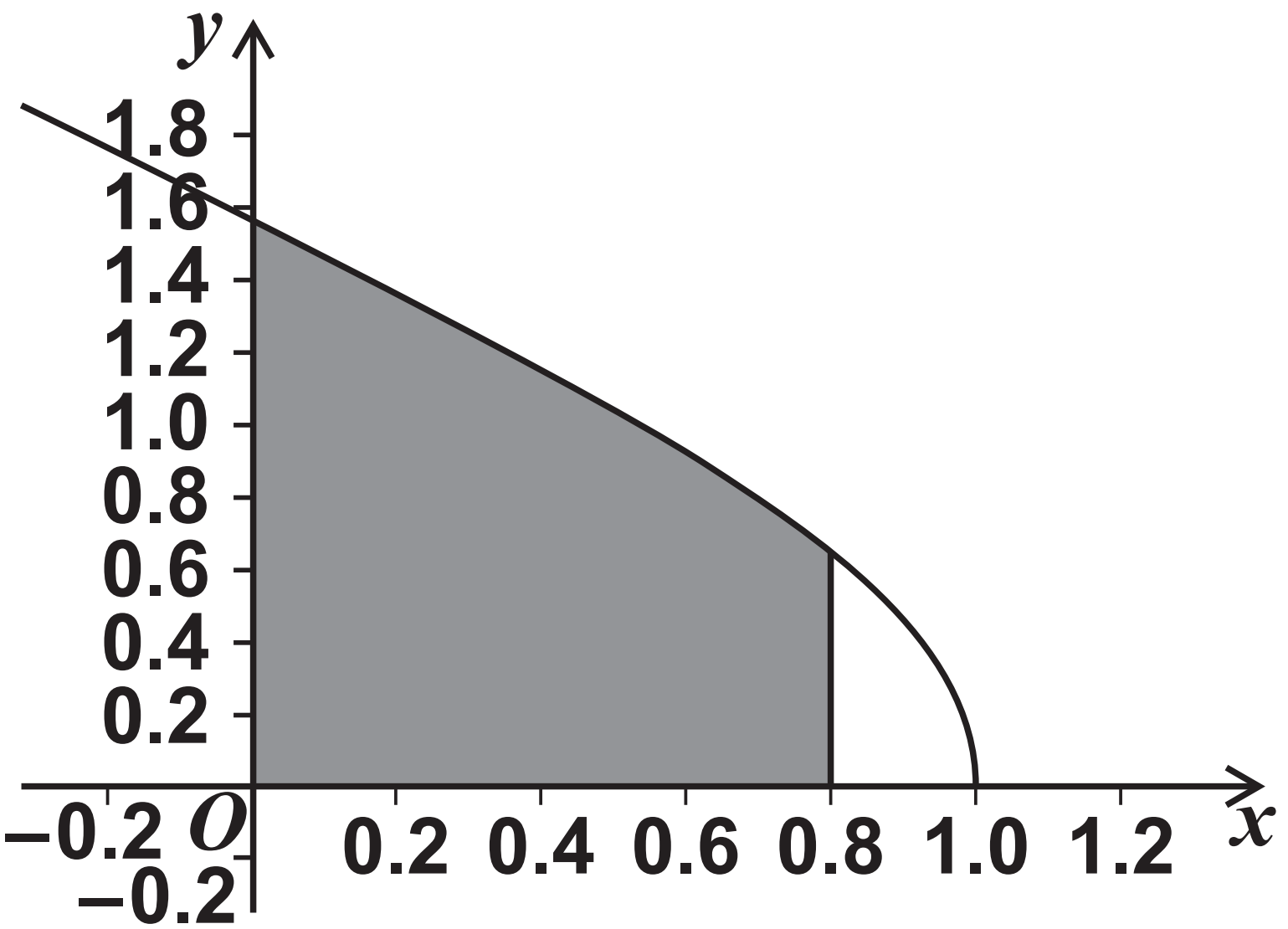
[Turn over]

7 The diagram shows part of the graph of $y = \cos^{-1} x$



The finite region enclosed by the graph of $y = \cos^{-1} x$, the y-axis, the x-axis and the line $x = 0.8$ is rotated by 2π radians about the x-axis.

Use Simpson's rule with five ordinates to estimate the volume of the solid formed. Give your answer to four decimal places. [5 marks]

[Turn over]

20

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[Turn over]

8 (a) **Factorise**

$$\begin{vmatrix} 2a+b+x & x+b & x^2+b^2 \\ 0 & a & -a^2 \\ a+b & b & b^2 \end{vmatrix}$$

as fully as possible. [6 marks]

[Turn over]

[Turn over]

8 (b) **The matrix M is defined by**

$$\mathbf{M} = \begin{bmatrix} 13 + x & x + 3 & x^2 + 9 \\ 0 & 5 & -25 \\ 8 & 3 & 9 \end{bmatrix}$$

Under the transformation represented by M, a solid of volume 0.625 m^3 becomes a solid of volume 300 m^3

Use your answer to part (a) to find the possible values of x.
[3 marks]

[Turn over]

9 The matrix $\mathbf{C} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, where a and b are positive real numbers,

and $\mathbf{C}^2 = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

Use C to show that $\cos\frac{\pi}{12}$ can be written in the form $\frac{\sqrt{\sqrt{m}+n}}{2}$, where m and n are integers.

[7 marks]

[Turn over]

[Turn over]

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[Turn over]

10 The sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 0 \qquad u_{n+1} = \frac{5}{6 - u_n}$$

Prove by induction that, for all integers $n \geq 1$,

$$u_n = \frac{5^n - 5}{5^n - 1}$$

[6 marks]

[Turn over]

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[Turn over]

11 (a) **Starting from the series given in the formulae booklet, show that the general term of the Maclaurin series for**

$$\frac{\sin x}{x} - \cos x$$

is

$$(-1)^{r+1} \frac{2r}{(2r+1)!} x^{2r}$$

[4 marks]

[Turn over]

11 (b) Show that

$$\lim_{x \to 0} \left[\frac{\frac{\sin x}{x} - \cos x}{1 - \cos x} \right] = \frac{2}{3}$$

[4 marks]

[Turn over]

12 (a) Given that $I = \int_a^b e^{2t} \sin t \, dt$, show that

$$I = [qe^{2t} \sin t + re^{2t} \cos t]_a^b$$

where q and r are rational numbers to be found. **[6 marks]**

[Turn over]

12(b) A small object is initially at rest. The subsequent motion of the object is modelled by the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} + v = 5\mathrm{e}^{t}\sin t$$

where v is the velocity at time t.

Find the speed of the object when $t = 2\pi$, giving your answer in exact form. [6 marks]

[Turn over]

13 **Charlotte is trying to solve this mathematical problem:**

Find the general solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + \frac{\mathrm{d}y}{\mathrm{d}x} - 2y = 10\mathrm{e}^{-2x}$$

Charlotte's solution starts as follows:

Particular integral: $y = \lambda \mathrm{e}^{-2x}$

so

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -2\lambda \mathrm{e}^{-2x}$$

and

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 4\lambda \mathrm{e}^{-2x}$$

13 (a) **Show that Charlotte's method will fail to find a particular integral for the differential equation.**
[2 marks]

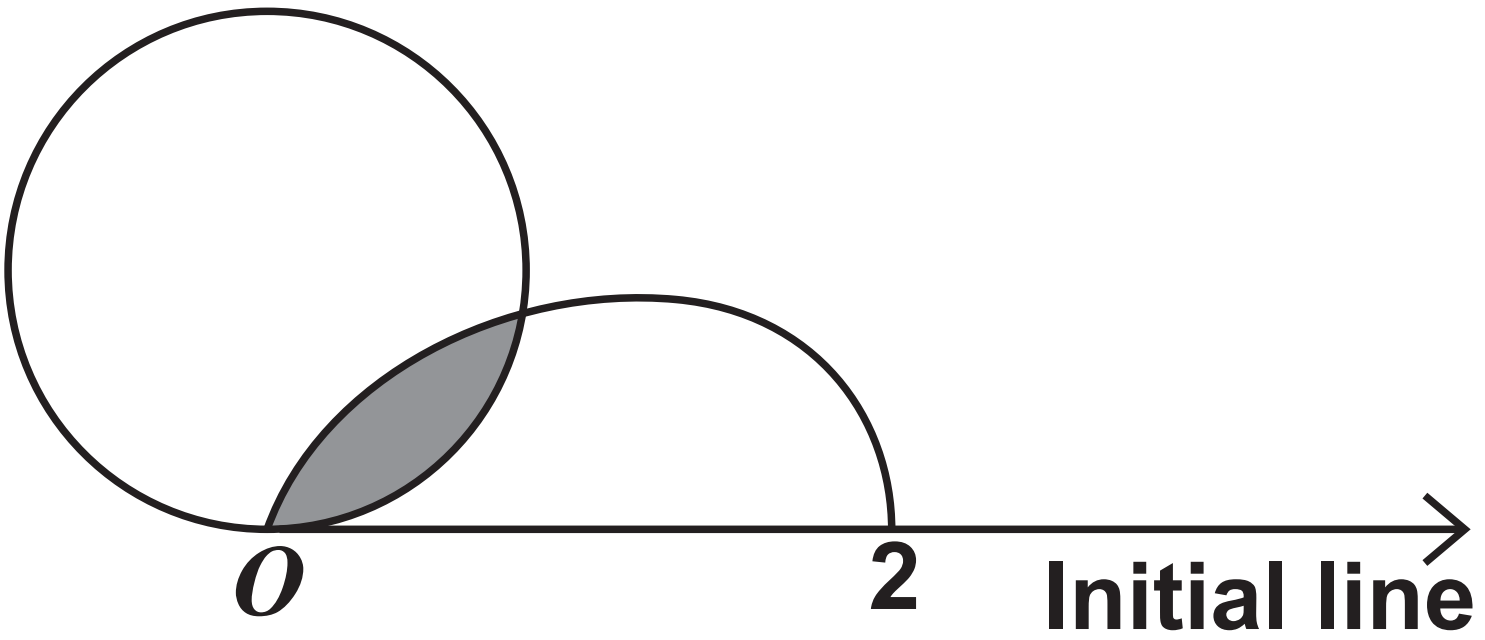
13(b) **Explain how Charlotte should have started her solution differently and find the general solution of the differential equation. [8 marks]**

[Turn over]

14 The diagram shows the polar curve C_1 with equation

$r = 2\sin\theta$

The diagram also shows part of the polar curve C_2 with equation $r = 1 + \cos 2\theta$



14 (a) On the diagram above, complete the sketch of C_2 [2 marks]

14(b) Show that the area of the region shaded in the diagram is equal to

$$k\pi + m\alpha - \sin 2\alpha + q \sin 4\alpha$$

where $\alpha = \sin^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$, and

k, m and q are rational numbers.
[9 marks]

[Turn over]

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[Turn over]

15 The points $A(7, 2, 8)$, $B(7, -4, 0)$ and $C(3, 3.2, 9.6)$ all lie in the plane Π.

15 (a) Find a Cartesian equation of the plane Π. [3 marks]

[Turn over]

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15 (b) The line L_1 has equation

$$\mathbf{r} = \begin{bmatrix} 5 \\ -0.4 \\ 4.8 \end{bmatrix} + \mu \begin{bmatrix} 15 \\ 3 \\ 4 \end{bmatrix}$$

15 (b) (i) Show that L_1 lies in the plane Π. **[2 marks]**

15 (b) (ii) Show that every point on L_1 is equidistant from B and C.
[4 marks]

[Turn over]

15 (c) The line L_2 lies in the plane Π, and every point on L_2 is equidistant from A and B.

Find an equation of the line L_2
[4 marks]

[Turn over]

15(d) The points *A*, *B* and *C* all lie on a circle *G*.
The point *D* is the centre of circle *G*.

Find the coordinates of *D*.
[3 marks]

END OF QUESTIONS

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For Examiner's Use	
Question	Mark
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